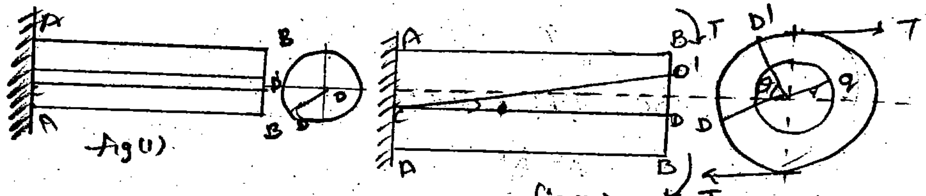
A 27/12/18

UNIT-2

Torsion of Shaft and Spring

A shaft is said to be in torsion when equal and opposite torque are apply at 2 ends of the shaft and the torque is equal to the product of force applied (Tangential at the ends of shaft) and radius of shaft due to the application of torque at 2 ends the shaft is subjected to twisting moment.

Derivation of shear stress produced in circular shaft subjected to torsion:-



fig(1) fig(2)

When the circular shaft is subjected to torsion shear stresses are developed in the material. Consider a shaft which is fixed at end AA and free at other end BB. mark a line CD on the outer surface of the shaft as shown in fig(1).

let 'T' be the torsion applied on the shaft at the end BB due to this torsional force the end BB rotates clockwise and the c/s of shaft the subjected to shear stresses as shown in fig(2) let R = radius of shaft

τ = shear stresses due to torque 'T'

L = length of the shaft

T = Torque applied

ϕ = $\angle DCD'$ shear strain

θ = $\angle DOD'$ which is nothing but angle of twist.

C = Modulus of rigidity (or) shear modulus.

Now, the distorsion (change) on the shaft due to torque (T) is DD'

from $\triangle^{le}$ DCD' $\tan\phi = \frac{DD'}{CD}$

when ϕ is very small 'tan' is neglected, then

wkt CD = length of shaft $= L$

$$\phi = \frac{DD'}{L}$$

Arc $DD' = R*\theta$

$$\phi = \frac{R\theta}{L}$$

Now, from modulus of rigidity, wkt

$$C = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi} \quad \left[\because \phi = \frac{R\theta}{L}\right]$$

$$C = \frac{\tau * L}{R\theta}$$

$$\tau = \frac{C \cdot R\theta}{L}$$

The above eqn can be written as

$$\frac{\tau}{R} = \frac{C\theta}{L}$$

observing the above eqn C, θ, & L constant & variable is R.

$\tau \propto R$

So, $\frac{\tau}{R}$ = constant.

radius 'r' having a shear stress q then,

$$\frac{q}{r} = \text{constant}$$

rearranging the above eqn

$$\frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r}.$$

Assumptions:-

→ The shaft is of same material

→ The C/S of the shaft is uniform.

→ All the radii are same before & after twist

→ The shaft is uniform before & after twist.

→ The twist along the shaft is uniform

* The max. shear stresses are developed at outer surface of the shaft and the min. shear stress in the shaft are zero at center.

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Max Torque Transmitted by Circular Shaft

Consider a shaft which is subjected to torque T as shown in fig.

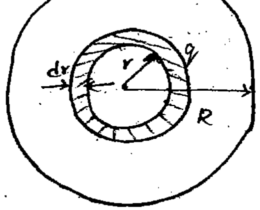
The max torque transmitted is obtained from the max. shear stress induced at outer surface.

let τ = max shear stress

R = radius of shaft

q = shear stress at radius 'r' from the Center

$= \frac{\tau}{R} * r$



fig(1)

From fig.1. Considering an Elementary ring at a radius 'r' from the center of dr thickness and dA area

We know that $\frac{\tau}{R} = \frac{q}{r}$

$$\therefore q = \frac{\tau}{R} * r$$

Now, the force acting on the Elimentary ring is shear stress acting into area of Elimentary ring

$$\therefore \text{Force} = q * dA$$
$$= \frac{\tau}{R} * r * 2\pi r * dr$$
$$= \frac{\tau}{R} 2\pi r^2 dr.$$

This is the force acting on Elimentary ring.

The turning moment or torque acting (dT) on ~~nothing but~~ the Elimentary ring is given by the product of the force acting on the Elimentary ring and distance of Elimentary ring from center.

$$dT = \frac{\tau}{R} * 2\pi r^2 dr * r$$

$$dT = \frac{\tau}{R} * 2\pi r^3 dr$$

Integrating dT on b/s by limits 0 to R

$$T = \frac{\tau}{R} * 2\pi \int_0^R r^3 dr$$
$$= \frac{\tau}{R} 2\pi \left[\frac{r^4}{4}\right]_0^R$$
$$= \frac{\tau}{R} 2\pi \left(\frac{R^4}{4}\right)$$
$$= \frac{\tau \pi R^3}{2} \qquad \because R = D/2$$
$$\therefore T = \frac{\tau \pi D^3}{16} \rightarrow \text{max torque}$$

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1. A solid shaft of 150mm dia is used to transmit the torque, find the max torque transmitted by the shaft if the max. shear stress induced in the shaft is $45 N/mm^2$.

Sol:

Given data.

d = dia of shaft = 150 mm

τ = shear stress = $45 N/mm^2$

$$\therefore \text{max torque} = \frac{45 * \pi * (150)^3}{16}$$

$$\therefore T = 29.82 \text{ MN-mm}^2 \text{ (KN-m)}$$

2. The shear stress of a solid shaft is not exceeding $40\ N/mm^2$ when the torque transmitted 20 kNm. Determine the min dia of the shaft.

Sol:

Given data.

$\tau = 40\ N/mm^2$

$T = 20\ kN\text{-}m = 20 \times 10^6\ N\text{-}mm$

WKT

$$T = \frac{\tau \pi D^3}{16}$$

$$20 \times 10^6 = \frac{40 \times \pi \times D^3}{16}$$

$$20 \times 10^6 = 7.85\ D^3$$

$$D^3 = 2{,}546{,}479.089$$

$\therefore D = 136.55$ mm

Max torque transmitted by hallow shaft

Consider a hallow shaft which is subjected to torque T.

Consider an elimentary ring of hallow shaft having dA. 'r' from center of thickness dr

Let, R_o = outer radius of the shaft

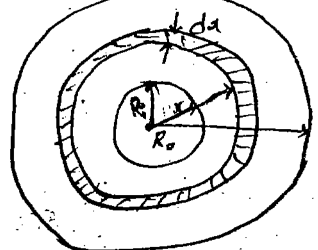
R_i = inner radius of the shaft

q = Shear stress induced in elimentary ring which is at a distance 'r' from center.

q

$\frac{\tau}{R} = \frac{q}{r}$, The max ... so

surface, Here the outer surface radius is R_o, Hence the above E^n becomes



$$\frac{\tau}{R_o} = \frac{q}{r}$$

$$q = \frac{\tau}{R_o} \times r$$

Shear stress induced in elimentary ring $= q$

$$\text{i.e., } q = \frac{\tau}{R_o} \times r$$

The force acting on the elimentary ring is given by product of shear stress acting and area of the elimentary ring.

$$\text{Force} = q \cdot dA$$

$$= \frac{\tau}{R_o} \times r \times 2\pi r \times dr$$

$$= \frac{\tau}{R_o} \times 2\pi r^2 \times dr$$

The turning moment or torque acting on the elimentary ring is given by product of force acting and distance of elimentary ring.

$$dT = \text{Force} \times \text{distance}$$

$$= \frac{\tau}{R_o} \times 2\pi r^2 \times dr \times r$$

$$= \frac{\tau}{R_o} \times 2\pi r^3 \times dr$$

Int dT on b/s by limits R_i to R_o

$$T = \frac{\tau}{R_o} * 2\pi \int_{R_i}^{R_o} r^3 \, dr$$

$$= \frac{\tau}{R_o} * 2\pi \left[\frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$= \frac{\tau}{R_o} 2\pi \left(\frac{R_o^4}{4} - \frac{R_i^4}{4}\right)$$

$$T = \frac{2\tau\pi}{R_o}\left[R_o^4 - R_i^4\right] \qquad \left[\because R_i = \frac{D_i}{2}\right]$$

$$= \frac{2\tau\pi}{\frac{D_o}{2}}\left(\frac{D_o^4}{16} - \frac{D_i^4}{16}\right) \qquad \left[R_o = \frac{D_o}{2}\right] \quad R_o^4 = \frac{D_o^4}{16}$$

$$\therefore T = \frac{\tau\pi}{16 D_o}\left[D_o^4 - D_i^4\right] \qquad R_i^4 = \frac{D_i^4}{16}$$

1. In a hallow circular shaft of outer & inner dias. are 20cm & 10cm respectively. The shear stress is not exceeding 40 N/mm². Determine the max torque for which the shaft can safely transmit.

Sol:

Given data:, $1\,cm = 10\,mm$

$D_o = 20\,cm = 200\,mm$

$D_i = 10\,cm = 100\,mm$

$\tau = 40\, N/mm^2$

$$T = \frac{\tau\pi}{16 D_o}\left[D_o^4 - D_i^4\right]$$

$$= \frac{40 * \pi}{16(200)}\left[(200)^4 - (100)^4\right]$$

$$= 58.90 \times 10^6 \, N\text{-}mm$$

Q. power transmitted by shaft

let T = torque acting on the solid shaft

The power transmitted by the shaft can be given as ωT

where $\omega = \frac{2\pi N}{60}$ = angular speed

N = no. of rotations per min in rpm

Example:

1. 2 shafts of same material & same length are subjected to same torque, if the 1st shaft is solid shaft and 2nd shaft is hallow shaft whose internal dia is equals to 2/3 of the outer dia and max shear stress developed in the each shaft is same. Compare the weights of the shafts.

Sol:

Given,

$L_S = L_H = L$ = length of shaft

D = Dia of solid shaft

D_o = outer dia. of hallow shaft

D_i = inner dia. of hallow shaft

W_S = wt. of solid shaft in kg

W_H = wt of Hollow shaft in kg; $\quad \therefore D_i = \frac{2}{3} D_o$

$\tau_S = \tau_H = \tau$

$\therefore W_S$ = bulk density * volume

$= \rho * vol$

$= \rho *$ Area * length

$\therefore W_S = \rho * \frac{\pi D^2}{4} * L$

$\therefore W_H = w * \text{Area} * \text{length}$

$= w * \frac{\pi}{4}(D_o^2 - D_i^2) * L$

$$\frac{W_S}{W_H} = \frac{D^2}{D_o^2 - D_i^2}$$

The shear stress of 2 shafts are same, so the torques of 2 shafts are also same

$\therefore T_S = T_H$

$$\frac{\pi \cancel{\tau}}{\cancel{16}} D^3 = \frac{\pi}{\cancel{16}} \tau \left(\frac{D_o^4 - D_i^4}{D_o}\right)$$

$$D^3 = \frac{D_o^4 - D_i^4}{D_o} \quad \left[\because D_i = \tfrac{2}{3} D_o\right]$$

$$= \frac{D_o^4 - \left(\frac{2}{3}D_o\right)^4}{D_o}$$

$$= \frac{D_o^4 - \frac{16}{81} D_o^4}{D_o}$$

$$D^3 = \frac{65}{81} D_o^3$$

$$= 0.93 D_o$$

Now

$$\therefore \frac{W_S}{W_H} = \frac{(0.93 D_o)^2}{D_o^2 - \left(\frac{2}{3}D_o\right)^2}$$

$$= \frac{(0.93)^2 \cancel{D_o^2}}{\cancel{D_o^2}\left(1 - \frac{4}{9}\right)}$$

$$\frac{W_S}{W_H} = \frac{1.55}{1}$$

2/1/11 A solid circular shaft and a hollow circular shaft whose inside dia is 3/4 of the outside dia. These 2 shafts are of same material & equal length and are required to transmit a given torque. Compare the wts of the 2 shafts, if the max shear stress developed in the 2 shafts are equal. HO

Sol:

$L_S = L_H = L$ = length of the shaft.

D = Dia of solid shaft.

D_o = Dia of hallow circular shaft.

D_i = Inner dia of hallow circular shaft.

$D_i = 3/4 \, D_o$

$\tau_S = \tau_H = \tau$

W_S = wt of solid shaft in kg.

$W_S = w * \frac{\pi D^2}{4} * L$

$W_H = w * \frac{\pi}{4}(D_o^2 - D_i^2) * L$

$$\frac{W_S}{W_H} = \frac{D^2}{D_o^2 - D_i^2}$$

$\therefore T_S = T_H$

$$\frac{\pi \cancel{\tau} D^3}{\cancel{16}} = \frac{\pi}{\cancel{16}} \tau \left(\frac{D_o^4 - D_i^4}{D_o}\right)$$

$$D^3 = \left(\frac{D_o^4 - D_i^4}{D_o}\right) \quad \left(D_i = \tfrac{3}{4} D_o\right)$$

$$= \frac{D_o^4 - \left(\frac{3}{4}D_o\right)^4}{D_o}$$

$$= \frac{D_o^4 - \frac{81}{256} D_o^4}{D_o}$$

$$= \frac{256 D_o^4 - 81 D_o^4}{256 D_o}$$

$$= \frac{175 D_o^4}{256 D_o}$$

$$D^3 = \frac{175}{256} D_o^3$$

$$D^3 = 0.68 D_o^3$$

$$D = 0.88 D_o$$

Now, $\frac{W_s}{W_H} = \frac{(0.88 D_o)^2}{D_o^2 - \left(\frac{3}{4} D_o\right)^2}$

$$= \frac{\cancel{D_o^2}(0.88)^2}{\cancel{D_o^2}\left(1 - \frac{9}{16}\right)}$$

$\therefore \frac{W_s}{W_H} = \frac{\cancel{3.0976}}{9} \; \frac{1.77}{1}$

3 A hallow shaft of External dia 180mm transmits 3,000 K-watts power at 200 rpm. Determine the max-internal dia if max. shear stress in the shaft is not exceeding 60 N/mm².

Given,

External dia = 120mm

ωT = 3,000 K-watts

N = 200 rpm

max shear stress τ_{max} = 60 N/mm²

$\omega T = 300$

$$T = \frac{300 \times 10^3}{\omega}$$

$$= \frac{3 \times 10^5}{\frac{2\pi N}{60}}$$

$$= \frac{3 \times 10^5}{\frac{2\pi(200)}{60}}$$

$$= \frac{3 \times 10^5}{20.94}$$

$T = 14326.64$ N-mm

$$T = \frac{\pi \tau}{16} \frac{(D_o^4 - D_i^4)}{D_o}$$

$$14326.64 = \frac{\pi \times 60}{16} \left(\frac{120^4 - D_i^4}{120}\right)$$

$$14326.64 = 11.780 (120^4 - D_i^4)$$

$$120^4 - D_i^4 = 1216.08$$

$$D_i^4 = 120^4 - 1216.08$$

D_i = 119.99 mm

$$300 \times 10^3 = \frac{2\pi \times 200}{60} \times T$$

T = 14.32 kN-mm

$D_o^4 - D_i^4 = 1215.85$

$= 11,780 \left(1728,000 - \frac{D_i^4}{120}\right)$

$1.215 \times 10^3 = 1728,00- \frac{D_i^4}{120}$

$\frac{D_i^4}{120} = 1727999.999$

$D_i = 88$

3|11|

Expression for torque in terms of polar m.o.I

polar moment of inertia is, the moment of inertia of an area about an axis ⊥ to the plane and passing through the centre of gravity of the area. It is denoted by "J".

$F = \tau \times A$
$= \frac{\tau}{R} \times r \times 2\pi r\,dr$

The torque interms of poI is obtained from

$dT = F \times rad$
$dT = \frac{\tau}{R} r \times 2\pi r \times r\,dr$

W.k.t $dT = \frac{\tau}{R} \times 2\pi r^3 \cdot dr$

$\therefore dA = 2\pi r * dr$

Hence $dT = \frac{\tau}{R} r^2 dA$ —— ①

But, $r^2 dA$ is nothing but p.M.O.I of the elimentary ring

Int Eq^n ① with limits 0 to R

$\therefore T = \frac{\tau}{R}\int_0^R r^2\,dA$ —— ②

From Eq^n ②, $\int_0^R r^2 . dA$ is p.M.O.I of circular shaft.

$\therefore T = \frac{\tau}{R} * J$

$\frac{T}{J} = \frac{\tau}{R}$, $\left[\because \frac{\tau}{R} = \frac{C\theta}{L}\right]$

So, we can write as,

$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$

T = torque
τ → shear stress
C → shear modulus
θ = angle of twist
J = PMOI
R = radius, L = Length

polar modulus

It is denoted by Z_p, it is defined as the ratio of PMO.I. to radius of the shaft. It is also known as torsional section modulus.

$\therefore$ For solid shaft $Z_p = \frac{J}{R}$

$= \frac{\frac{\pi}{32} D^4}{D/2}$

$= \frac{\pi}{16} D^3$

$\therefore$ For hallow shaft $Z_p = \frac{J}{R}$

$= \frac{\frac{\pi}{16}(D_o^4 - D_i^4)}{D_o}$

$= \frac{\pi [D_o^4 - D_i^4]}{16 D_o}$

Strength of shaft and torsional rigidity

It means the max. torque or the max. power that can be transmitted by shaft.

The torsional rigidity or stiffness of the shaft is defined as the product of modulus of rigidity and polar moment of inertia.

$\frac{T}{J} = \frac{C\theta}{L}$
$\frac{T}{J} = C$
$T = C * J$

i.e., $T = C * J$

The torsion rigidity is also defined as, the torque required to produce twist of one radian per unit length

Example

1. Determine the dia. of solid shaft which will transmit 90 kW at 160 rotation per min. also determine the length of shaft, if the twist must not exceed 1° over the entire span. The max. shear stress is limited to 60 N/mm², shear modulus is $8*10^4$ N/mm²

Sol:

Given data, $\tau = 60$

dia = ?

$P = 90$ kW

$N = 160$ rpm

$C = 8*10^4$ N/mm²

angular speed, $\omega = \frac{2\pi N}{60}$

$= \frac{2\pi * 160}{60}$

$= 16.75$

$T = ?$

$P = T*\omega$

$T = \frac{P}{\omega}$

$= \frac{90}{16.75}$

Torque $T = 5.73$ kN-m

$T = \frac{\pi}{16} \tau D^3$

$5.73 = \frac{\pi}{16} 60 D^3$

$\therefore D = 76.95$ mm

length = ?

$\frac{\tau}{R} = \frac{C\theta}{L}$

$\frac{60}{\frac{76.95}{2}} = \frac{8*10^4 * \pi}{180 * L}$

$1.55 = \frac{1396.26}{L}$

$L = 895.03$ mm

2. Determine the dia. of solid shaft which will transmi[t] 300 kW at 250 rpm and the max. shear stress is not exceeding 30 N/mm² and angular twist is should not be more than 1° in a shaft of length 2m, take shear modulus as $1*10^5$ N/mm².

Sol:

Given data

$P = 300$ kW

$N = 250$ rpm

$L = 2$m

$\theta = 1° * \frac{\pi}{180} = 0.01745$

$\tau = 30$ N/mm²

$C = 1*10^5$ N/mm²

$T = \frac{\pi}{16} \tau D^3$

$\omega = \frac{2\pi N}{60} = \frac{2*\pi*250}{60} = 26.17$

$P = T\omega$

$T = \frac{P}{\omega} = \frac{300}{26.17} = 11.46$ kN-m

$11.46 \times 10^6 = \frac{\pi}{16} \times 30 \times D^3$

$D^3 = 1945,510.024$

$D = 124.84 \simeq 125$

$D = 125\,mm$.

$\frac{\tau}{R} = \frac{C\theta}{L}$

$\frac{30}{D/2} = \frac{1 \times 10^5 \times \pi}{180 \times 2 \times 10^3}$

$\frac{30}{D/2} = 0.8726$

$D = \underline{68.76}\ mm$

3. A hallow shaft of ~~dia ratio~~ $\frac{dia}{3m}$ ratio 3/8 - internal to outer dia, is to transmit 375 kW power at 100 rpm and the max. torque is being 20% greater then the mean torque. The shear stress is not exceeding $60\,N/mm^2$ and the twist is not exceeding $2°$ over the span of 4m. Calculate both inner dia & outer dia for which it would satisfy both the conditions. Take shear modulus $C = 0.85 \times 10^5\,N/mm^2$.

Sol:

Given.

dia, $D_i = \frac{3}{8} D_0$

$P = 375\,kW$

$\tau = 60\,N/mm^2$

$\theta = 2° \times \frac{\pi}{180} = 0.0349$

$N = 100$ rpm

$L = 4m$

$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 100}{60} = 10.472$

$T_{mean} = \frac{P}{\omega} = \frac{375}{10.472} = 35.80\ kN-m$

$\rightarrow T_{max} = 1.2\,T_{mean}$
$= 1.2 \times 35.8$
$= 42.96\ kN-m$

$\therefore T = \frac{\tau \pi}{16 D_0}(D_0^4 - D_i^4)$

$42.97 \times 10^6 = \frac{\tau \pi}{16 D_0}\left(D_0^4 - \left(\frac{3}{8} D_0\right)^4\right)$

$= \frac{60 \times \pi}{16\,D_0}\left(D_0^4 - \frac{81}{4096} D_0^4\right)$

$= \frac{60 \times \pi}{16 D_0}\left[\frac{(4096 - 81) D_0^4}{4096}\right]$

$= \frac{60 \times \pi}{16}\left(\frac{4015\,D_0^3}{4096}\right)$

$= 11.780\,(0.9802\,D_0^3)$

$42.97 \times 10^6 = 11.5477\,D_0^3$

$\therefore D_0 = 155\,mm$

$D_i = \frac{3}{8}(155)$

$\therefore D_i = 58.125\,mm$

Condition-2

$\frac{\tau}{R} = \frac{C\theta}{L}$

$\frac{60}{D/2} = \frac{0.85 \times 10^5 \times 0.0349}{4}$

$\frac{60}{D_0 - D_i} =$

$\frac{6}{D_0 - D_i} = 9.88$

$D_0 - D_i = 1.1012$

$D_0 - \frac{3}{8} D_0 = 1.1012$

$\frac{5}{8} D_0 =$

Combine bending moment and torsion

We know that when a shaft is transmitting torque it is subjected to shear stresses. At the same time the shaft is also subjected to moments due to gravity loads. Due to the B.M. the bending stresses are developed in the shaft, so for design purpose it is necessary to find the p. stresses, max. shear stress & strain energy for the combination of bending stress & shear stress.

Consider any point on the c/s of shaft,

let T = torque at that s/c.

M = B.M at that s/c.

D = Dia of shaft

q = shear stresses developed due to torque.

σ = Bending stresses due to B.M.

∴ The shear stresses at any s/c can be given by, $\left[\frac{\tau}{R} = \frac{q}{r} = \frac{C\theta}{L} = \frac{T}{J}\right]$

$$\frac{q}{r} = \frac{T}{J}$$

$$q = \frac{T}{J} * r$$

∴ The Bending stresses at any s/c is given by

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} * y$$

∴ WKT, the angle made by a plane for a given stresses & shear stresses then,

$$\tan 2\theta = \frac{2\tau}{\sigma}$$

$\tau = \frac{T}{J} * R$, $\quad \sigma = \frac{M}{I} * y$ 45

$R = D/2$

$y = D/2$

$I = \frac{\pi}{64} D^4$

$J = \frac{\pi}{32} D^4$

$$\tan 2\theta = \frac{2\left(\frac{T}{J} * R\right)}{\left(\frac{M}{I} * y\right)}$$

$$= \frac{2\left(\frac{T\,32}{\pi D^4} * \frac{D}{2}\right)}{\left(\frac{M\,64}{\pi D^4} * \frac{D}{2}\right)}$$

$$\tan 2\theta = \frac{\frac{32T}{\pi D^3}}{\frac{32M}{\pi D^3}} = \frac{T}{M}$$

∴ Major principle stress is

$$= \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{M * y}{2I} + \sqrt{\left(\frac{M * y}{2I}\right)^2 + \left(\frac{T}{J} * R\right)^2}$$

$$= \frac{M * D/2}{2 \frac{\pi}{64} D^4} + \sqrt{\left(\frac{M * D/2}{2 * \frac{\pi}{64} D^4}\right)^2 + \left(\frac{T}{\frac{\pi}{32} D^4} * \frac{D}{2}\right)^2}$$

$$= \frac{M}{64 D^3} + \left(\frac{M}{64 D^3}\right)^2 + \left(\frac{16 T}{\pi D^3}\right)^2$$

$$= \frac{16 M}{\pi D^3} + \sqrt{\left(\frac{16 M}{\pi D^3}\right)^2 + \left(\frac{16 T}{\pi D^3}\right)^2}$$

$$= \frac{16 M}{\pi D^3} + \left[\frac{16}{\pi D^3} \sqrt{M^2 + T^2}\right]$$

$$= \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$$

Minor principle stress $= \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$

max shear stress $= \dfrac{\frac{16}{\pi D^3}(M+\sqrt{M^2+T^2}) - \frac{16}{\pi D^3}(M-\sqrt{M^2+T^2})}{2}$

$= \frac{1}{2}\,\frac{16^8}{\pi D^3}\left(M+\sqrt{M^2+T^2}-M+\sqrt{M^2+T^2}\right)$

$= \frac{8}{\pi D^3}\left(2\sqrt{M^2+T^2}\right)$

$= \frac{16}{\pi D^3}\left(\sqrt{M^2+T^2}\right)$

For hallow shaft.

Major p. stress $= \frac{M*y}{2I} + \sqrt{\left(\frac{M*y}{2I}\right)^2 + \left(\frac{T}{J}*R\right)^2}$

$= \dfrac{M * D_o/2}{2\,\frac{\pi}{64}(D_o^4-D_i^4)} + \sqrt{\left(\dfrac{M*D_o/2}{2*\frac{\pi}{64}(D_o^4-D_i^4)}\right)^2 + \left(\dfrac{T}{\frac{\pi}{32}(D_o^4-D_i^4)}*\frac{D_o}{2}\right)^2}$

$= \frac{16 D_o M}{\pi(D_o^4-D_i^4)} + \sqrt{\left[\frac{16 D_o M}{\pi(D_o^4-D_i^4)}\right]^2 + \left(\frac{16\,T\,D_o}{\pi(D_o^4-D_i^4)}\right)^2}$

$= \frac{16 D_o M}{\pi(D_o^4-D_i^4)} + \left(\frac{16 D_o}{\pi(D_o^4-D_i^4)}\sqrt{M^2+T^2}\right)$

$= \frac{16 D_o}{\pi(D_o^4-D_i^4)}\left(M+\sqrt{M^2+T^2}\right)$

Minor principle stress

$= \frac{16 D_o}{\pi(D_o^4-D_i^4)} * \left(M-\sqrt{M^2+T^2}\right)$

Max shear stress

$= \dfrac{\frac{16 D_o}{\pi(D_o^4-D_i^4)}(M+\sqrt{M^2+T^2}) - \frac{16 D_o}{\pi(D_o^4-D_i^4)}(M-\sqrt{M^2+T^2})}{2}$

$= \frac{16 D_o}{\pi(D_o^4-D_i^4)}\left(\sqrt{M^2+T^2}\right)$

1. Solid shaft of dia 80 mm is subjected to a twisting moment of 8 MN-mm & Bending moment of 5 MN-mm. Determine, 1. principle stresses
2. The position of the plane on which they act.
3.

Sol:

Given,
Dia $D = 80$ mm
$T = 8$ MN-mm
$M = 5$ MN-mm

1. Major principle stress,

$= \frac{16}{\pi D^3}\left(M+\sqrt{M^2+T^2}\right)$

$= \frac{16}{\pi * 80*10^3}\left(5*10^6 + \sqrt{(5*10^6)^2 + (8*10^6)^2}\right)$

$= 143.5$ N/mm²

Minor p. str

$= \frac{16}{\pi D^3}\left(M-\sqrt{M^2+T^2}\right)$

$= \frac{16}{\pi * 80*10^3}[5 -$

2. position of plane $\tan 2\theta = \frac{T}{M}$

$= \frac{8}{5}$

$= 1.6$

$\theta = 28^\circ 59'$

2. The max allowable shear stress in a hollow shaft of external dia is twice the internal dia is 80 N/mm². Determine the dia of shafts, if it is subjected to torque of 4 MN-mm and Bending moment of 3 MN mm

Sol: Given,

Max shear stress $\tau = 80\ N/mm^2$

$D_o = 2D_i$

$T = 4\ MN\text{-}mm$

$M = 3\ MN\text{-}mm$

Max shear stress $= \frac{16 D_o}{\pi(D_o^4 - D_i^4)} \sqrt{M^2 + T^2}$

$80 = \frac{16(2D_i)}{\pi((2D_i)^4 - D_i^4)} \sqrt{3^2 + 4^2}$

$= \frac{32 D_i}{\pi(16D_i^4 - D_i^4)} \times 5$

$= \frac{160 D_i}{\pi \cdot 12 D_i^4}$

$80 = \frac{160}{37.69 D_i^3}$

$D_i^3 = \frac{4.211}{80}$

$D_i^3 = 0.052$

$= 0.374\ mm$

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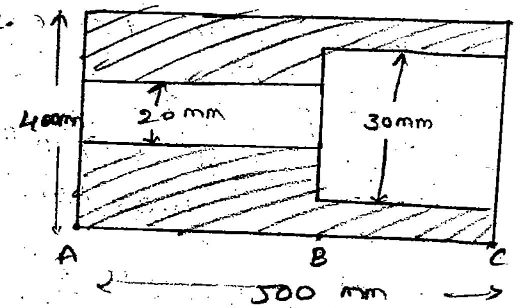
Strength of shaft of varying section

When a shaft is made up of different lengths and of different diameters, the torque transmitted by the individual shafts should be calculated first and the strength of shaft is the min. value of these shafts.

Example

1. A shaft ABC of 500mm length and 40mm External dia is bored for a part of length AB to a 20mm dia and for the remaining part BC 30mm dia, If the shear stresses developed is not exceeding 80 N/mm². Find the max power the shaft can transmit at a speed of 200 rpm. If angle of twist is same find the length of the shaft that has been bored to 20mm & 30mm dia.



Sol: Given data

$\tau = 80\ N/mm^2$

$N = 200$ rpm

$D_o = 40$ mm

Shaft $AB = D_i = 20mm$

$BC = D_i = 30mm$

Let $\theta_1 = \theta_2 = \theta$

Let length $AB = L_1$, $BC = L_2$

To find

$P = ?$

$L_1 = ?$

$L_2 = ?$

$P = T\omega$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94$

T = smaller of T_1 & T_2

T_1 = torque in AB

$= \frac{\tau \pi}{16 D_o}(D_o^4 - D_i^4)$

$= \frac{80 \times \pi}{16(40)}(40^4 - 20^4)$

$T_1 = 942.47$ N-m

T_2 = torque in BC

$= \frac{80 \times \pi}{16(40)}(40^4 - 30^4)$

$\therefore T_2 = 687.22$ N-m

$\therefore T_2$ is smaller Hence $T = 687.22$ Nm

$\therefore P = T\omega$

$= 687.22 \times 20.94$

$= 14.38$ kW

From, $\frac{T}{J} = \frac{C\theta}{L}$ $(\because \theta_1 = \theta_2)$

$\theta = \frac{T}{J} \times \frac{L}{C}$ $(\because C = \text{constant})$

$\theta_1 = \theta_2$ $(\because T_1 = T_2)$

$\frac{T_1}{J_1} \times \frac{L_1}{C} = \frac{T_2}{J_2} \times \frac{L_2}{C}$

$\frac{\cancel{T_1}}{J_1} \times \frac{L_1}{\cancel{C}} = \frac{\cancel{T_2}}{J_2} \times \frac{L_2}{\cancel{C}}$

$\frac{L_1}{J_1} = \frac{L_2}{J_2}$

$\therefore J_1 = \frac{\pi}{32}(D_o^4 - D_i^4)$

$= \frac{\pi}{32}(40^4 - 20^4)$

$= 235.6 \times 10^3 \text{ mm}^4$

$\therefore J_2 = \frac{\pi}{32}(D_o^4 - D_i^4)$

$= \frac{\pi}{32}(40^4 - 30^4)$

$= 171.80 \times 10^3 \text{ mm}^4$

$\therefore L_1 = \frac{L_2}{J_2} \times J_1$

$= L_2 \frac{235.6}{171.80}$

$\therefore L_1 = 1.371 L_2$

$L_1 + L_2 = 500$

$1.371 L_2 + L_2 = 500$

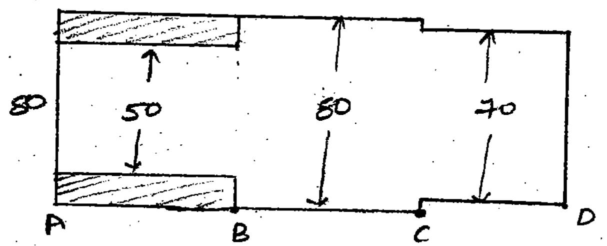
$2.371 L_2 = 500$

$\therefore L_2 = 210.88$ mm

$\therefore L_1 = 289.12$ mm

2. A steel shaft ABCD having a length of 2.4m Containing 3 lengths having different sections has follows. AB is hallow having outside & inside dia are 80mm & 50mm respectively and BC & CD are solid shafts which are BC having 80mm diameter and CD having 70mm dia. If the angle of twist is same in each section determine the length of each section and the total angle of twist if the max. shear stress in the hallow shaft is 50N/mm². Take shear modulus is $8.2*10^4$ N/mm².

Sol:



Given data,

$\tau_H = 50$ N/mm²

$C = 8.2*10^4$ N/mm²

Let L_1 = length of shaft AB

J_1 = p.M.O.I of AB shaft.

θ_1 = angle of twist of AB shaft.

L_2 = length of BC

J_1 = P.M.O.I of BC

θ_1 = θ of BC

L_3 = length of CD

J_1 = p.M.O.I of CD

θ_3 = θ of CD

$\therefore \theta = \theta_1 = \theta_2 = \theta_3$

$L_1 + L_2 + L_3 = 2.4\,m$

To find: $L_1 = ?,\ L_2 = ?,\ L_3 = ?,\ \theta = ?$

We know the relation,

$$\frac{T}{J} = \frac{C\theta}{L}$$

AB shaft,

$$\frac{T}{J_1} = \frac{C*\theta_1}{L_1}$$

$$J_1 = \frac{\pi}{32}(D_o^4 - D_i^4)$$

$$= \frac{\pi}{32}[80^4 - 50^4]$$

$$= 3.4*10^6\ mm^4$$

$$T = \frac{\pi}{16}\tau(D_o^4 - D_i^4)$$

$$= \frac{50*\pi}{16*80}(80^4 - 50^4)$$

$$\therefore T = 4.25*10^6\ N\text{-}mm$$

$$\therefore \theta_1 = \frac{T}{J_1}*\frac{L_1}{C}$$

$$\theta_1 = \frac{4.25*10^6}{3.4*10^6}*\frac{L_1}{8.2*10^4}$$

$$= 1.52*10^{-5} L_1$$

BC shaft,

$$J_2 = \frac{\pi}{32}D^4 = \frac{\pi}{32}*80^4$$

$$= 4.02*10^6\ mm^4$$

$$T = 4.25*10^6\ N\text{-}mm$$

$$\theta_2 = \frac{4.02*10^6}{4.25*10^6}*\frac{L_2}{8.2*10^4} = 1.15*10^{-5} L_2$$

$J_3 = \frac{\pi}{32} D^4$

$= \frac{\pi}{32} (70^4)$

$= 2.35 \times 10^6 \, mm^4$

$T = 4.25 \times 10^6 \, Nmm$

$\theta_3 = \frac{\cancel{2.35 \times 10^6}}{\cancel{4.25 \times 10^6}} * \frac{\cancel{L_3}}{\cancel{8.2 \times 10^4}}$ $\quad \frac{4.25 \times 10^6}{2.35 \times 10^6} \times L_3 = 8.2 \times 10^4 \times \theta_3$

$\theta_3 = 6.74 \times 10^{-6}$ $\quad \theta_3 = 2.205 \times 10^{-5} L_3$

$\theta = \theta_1 = \theta_2 = \theta_3$

$1.52 \times 10^{-5} L_1 = 1.288 \times 10^{-5} L_2 = 2.205 \times 10^{-5} L_3$

$L_1 = 0.847 L_2$

$L_1 = 1.450 L_3$

$L_3 = 0.584 L_2$

$L_1 + L_2 + L_3 = 2.4$

$0.847 L_2 + L_2 + 0.584 L_2 = 2.4$

$L_2 = 987.65 \, mm$

$L_1 = 836.53 \, mm$

$L_3 = 575.82 \, mm$

$\frac{T}{J} = \frac{C\theta}{L}$

$\theta = 0.0127$ Radians

$\underline{4.25 \times 10^6}$

Strain Energy stored in body due to torsion

Consider a solid shaft which is in torsion of radius R take an elementary ring of thickness "dr" which is at a distance of r from the center.

let, T = torque on shaft

R = radius of the shaft

τ = shear stress on the outer surface R.

q = shear stress in the elementary ring at 'r'

C = shear modulus

U = strain energy stored in the shaft.

w.k.t $\frac{q}{r} = \frac{\tau}{R}$

$\therefore$ shear stress in elementary ring can be written as

$$q = \frac{r * \tau}{R}$$

let dA = Area of elementary ring

$= 2\pi r \, dr$

V = volume of elementary ring

$= dA \cdot L$

The strain energy stored in elementary ring

$= \frac{q^2}{2C} * V$

$= \frac{\left[\frac{r}{R} * \tau\right]^2 2\pi r \, dr \, L}{2C}$

$= \frac{\tau^2}{2R^2 C} r^2 \, 2\pi r \, dr \, L$

$$= \frac{\tau^2 L}{2R^2 c} r^2 dA$$

strain energy stored in shaft with '0' to R

$$u = \int_0^R \frac{\tau^2 L}{2R^2 C} r^2 dA.$$

$$u = \frac{\tau^2 L}{2R^2 c} \int_0^R r^2 dA$$

$$u = \frac{\tau^2 L}{2R^2 c} J \quad \text{——} \quad (1)$$

but $J = \frac{\pi}{32} D^4$

$$u = \frac{\tau^2 L}{2R^2 c} * \frac{\pi}{32} D^4.$$

$$u = \frac{\tau^2 L}{2R^2 c} * \frac{\pi}{32} (16 R^4) \qquad (\because D = 2R)$$

$$= \frac{\tau^2 L}{4C} R^2 \pi$$

$$u = \frac{\tau^2}{C} * \frac{\pi}{4} R^2 L \qquad (\pi R^2 = \text{Area})$$

$$u = \frac{\tau^2}{4c} * v$$

For hollow shaft :- $J = \frac{\pi}{32}(D^4 - d^4)$

Sub J in Eq^n (1)

$$u = \frac{\tau^2}{2R^2 c} L\left(\frac{\pi}{32}(D^4 - d^4)\right]$$

$$u = \frac{\tau^2 L}{2R^2 c} \left[\frac{\pi}{32}(D^2 + d^2)(D^2 - d^2)\right]$$

But $R = D/2$

$$u = \frac{2\tau^2 L}{D^2} \left[\frac{\pi}{32}(D^2 - d^2)(D^2 + d^2)\right]$$

$$= \frac{\tau^2 L}{4D^2 c} \frac{\pi}{4} \left[(D^2 - d^2)(D^2 + d^2)\right]$$

$$\cdot u = \frac{\tau^2}{4D^2 c} (D^2 + d^2) * v \qquad \left[\frac{\pi}{4}(D^2 - d^2) L = v\right]$$

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1. Determine the max strain energy stored in a body which is made up of, solid shaft dia 10cm and length 1.25m. If max allowable shear stress is 50 N/mm². Take shear modulus is $8*10^4$ N/mm².

Sol:- Given data.

$= 8*10^4$ N/mm²

dia D = 10 cm = 100mm

max strain energy stored in a solid shaft is = ? = $u = \frac{\tau^2}{4c} * v$

L = length of shaft = 1.25m = 1250 mm

τ = 50 N/mm²

$$u = \frac{(\tau)^2}{4c} * v$$

$$= \frac{(50)^2}{4*8*10^4} * \left[\frac{\pi}{4}(100)^2\right](1250)$$

u = 76.9 kN-mm

2. Calculate the dia's of hallow shaft of same length & same c/s area as a solid shaft of 15 cm dia. If the strain energy stored in the hallow shaft is 25% greater than the strain energy stored in solid shaft transmitting same torque at the same max shear stress.

Sol:- Given data,

let s_1 = solid shaft

s_2 = hallow shaft

The 2 shafts have same length and same cross sectional area

$\therefore L_1 = L_2 = L$

$\therefore A_1 = A_2 = A$.

If the length & Areas are same hence volume is also same

$V_1 = V_2 = V$

let D_1 = External dia

D_2 = internal dia

solid shaft dia D_S = 15 cm = 150 mm

Torque of 2 shafts are same, $T_1 = T_2 = T$

max shear stress also same to 2 shafts,

$\tau_1 = \tau_2 = \tau$

$\therefore$ For hallow shaft the max strain Energy is

$$U_h = \frac{\tau^2}{4D_1^2 C} V (D_1^2 + D_2^2)$$

$\therefore$ For solid shaft the max. strain energy

$$U_S = \frac{\tau^2}{4C} * V$$

$\therefore$ The max strain energy in hallow shaft is 25% > max strain energy of solid shaft

$U_h = 1.25 U_S$

$$\frac{\tau^2}{4D_1^2 C} V (D_1^2 + D_2^2) = 1.25 \frac{\tau^2}{4C} * V$$

$$D_1^2 + D_2^2 = 1.25 D_1^2$$

$$D_2^2 = 0.25 D_1^2$$

$D_2 = 0.5 D_1$

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The C/s of 2 shafts are same

$A_1 = A_2$

$$\frac{\pi}{4}(D_S^2) = \frac{\pi}{4}(D_1^2 - D_2^2)$$

$$D_S^2 = D_1^2 - D_2^2$$

$$150^2 = D_1^2 - D_2^2$$

$$2.25 * 10^4 = D_1^2 - (0.5 D_1)^2$$

$$= D_1^2 - 0.25 D_1^2$$

$$= 0.75 D_1^2$$

$$D_1^2 = \frac{2.25 * 10^4}{0.75}$$

$\therefore D_1 = 173.2$ mm

$\therefore D_2 = 86.6$ mm

3. A solid circular shaft of 10 cm dia of length 4m is transmitting 112.5 KW power at 150 rpm. Determine the max shear stress induced in the shaft.

1. The max. shear stress induced in the shaft.
2. strain Energy stored in the shaft.

Take $C = 8 * 10^4$ N/mm².

Sol:

Given data

dia D = 10 cm = 100 mm

P = 112.5 KW

N = 150 rpm

$C = 8 * 10^4$ N/mm²

length L = 4m = 4*1000 mm

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2 * \pi * 150}{60} = 15.70$$

$$P = T\omega$$

$$T = \frac{P}{\omega} = \frac{11205}{15.70} = 7.16*10^3$$

$$\frac{\pi q D^3}{16} = 7.16*10^3$$

$$\frac{\pi q (100)^3}{16} = 7.16*10^3$$

$$196.34*10^3 q = 7.16*10^3$$

$$q = \frac{7.16*10^3}{196.34*10^3}$$

$$= 0.036 \ N/mm^2$$

$$U = \frac{q^2}{4C} * V$$

$$= \frac{(0.036)^2}{4*8*10^4} \left[\frac{\pi}{4}(100)^2\right](4000)$$

$$=$$

Spring

Springs are the elastic bodies which absorb Energy due to ~~resolence~~ resilience. 53

This absorbed energy may be released as & when required

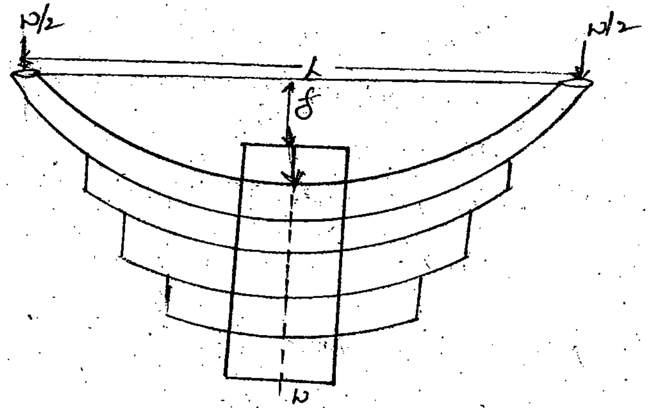
A spring which is capable of absorbing the greatest amount of energy for the given stress without getting permanently distorted is known as spring.

Their are 2-types of springs.

1. Laminated or leaf spring
2. Helical spring.

Laminated or leaf spring

These are used to absorb Energy in railway wagons, coaches and road vehicles as shown in fig.



let us consider a laminated spring which consist of no. of ‖le ~~springs~~ strips of a metal having different length & Same width placed one over the other as shown in fig.

10/1/19

Expression for max. bending stresses developed in plate

~~let us consider a laminated s~~

Initially all the plates are bent to the Same radius and are free to slide one over other which having the central deflection δ. this spring rests on the axis of vehicle and its top plate is connected to the chasis of the vehicle.

When the spring is loaded to the designed load "W". All the plates become flat and the central deflection δ disappears.

let, b = width of Each plate

n = no. of plates

l = span of spring

σ = max. bending stresses developed in the plate

W = point load acting at the center of span.

Moment of force $M = \frac{W}{2} * \frac{L}{2} = \frac{WL}{4}$

MOI of each plate $I = \frac{bt^3}{12}$

W.K.t, $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma = \frac{M}{I} * y$$

$$= \frac{\frac{WL}{4}}{\frac{bt^3}{12}} * \frac{t}{2}$$

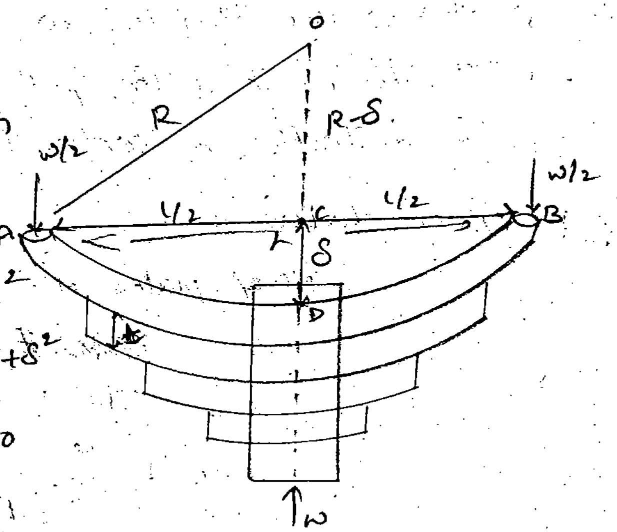
$$= \frac{\cancel{12}^{3} WL}{\cancel{4} bt^3} * \frac{t}{2}$$

$$= \frac{3WLt}{2bt^3}$$

the bending stress developed in one plate $\sigma = \frac{3WL}{2bt^2}$

For n no. of plates $\sigma = \frac{3}{2n}\frac{WL}{bt^2}$

Expression for central deflection of laminated spring



From the diagram

Δ^{le} ACO

$AO^2 = AC^2 + OC^2$

$R^2 = \left(\frac{L}{2}\right)^2 + (R-\delta)^2$

$\cancel{R^2} = \frac{L^2}{4} + \cancel{R^2} - 2\delta R + \delta^2$

$\frac{L^2}{4} - 2\delta R + \delta^2 = 0$

W.K.t

$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

$R = \frac{y}{\sigma} * E$

$R = \frac{Ey}{\sigma}$ $\qquad y = t/2$

$R = \frac{Et}{2\sigma}$ $\quad R = \frac{I}{M} * E$

$\therefore \frac{c^2}{4} - 2\delta\left(\frac{Et}{2\sigma}\right) - \delta^2 = 0$

neglecting δ for small value.

$\frac{c^2}{4} = 2\delta\left(\frac{Et}{2\sigma}\right)$

$\delta = \frac{L^2}{4} * \frac{\sigma}{Et}$

Example :-

1\. A Laminated spring carries a central load of 3 kN and the spring is made up of 10 steel plates of 5cm wide and 6 mm thick. If the bending stresses are limited to 150 N/mm². Determine

a. The length of the spring.

b. deflection at center of the spring.

Sol: Given data.

No of plates $N = 10$

~~width~~ t = 6mm

wide b = 5cm

$\sigma = 150$ N/mm²

load $w = 3$ kN

$\sigma = \frac{3}{2}\frac{wL}{bt^2}$

$150 = \frac{3}{2} * \frac{3 \times 10^3 L}{50(6^2)}$

$150 = 25L$

$L = \frac{150}{25} = 6$

$E = 2 \times 10^5$

$\delta = \frac{L^2}{4} * \frac{\sigma}{Et}$

$= \frac{6^2}{4} * \frac{150}{2 \times 10^5 \cdot 6}$

$\delta = 1.125 \times 10^{-3}$ mm

11/1/19 A laminated spring of 1m long is made up of plates each of 5cm wide and 1cm thick. If the bending stresses in the plates are limited to 100 N/mm². How many plates would be required to enable the spring to carry a central point of 2kN. If modulus of rigidity is 2.1×10^5 N/mm². what is the deflection under the load.

1m = 100 cm

Sol: Given data,

length $l = 1$m

wide b = 5cm = 0.05m

t = 1cm = 0.01m

$\sigma = 100$ N/mm²

$N = ?$

point load $w = 2$ kN

$E = 2.1 \times 10^5$ N/mm²

$\delta = ?$

$\sigma = \frac{3}{2}\frac{wL}{bt^2}$ fore one plate.

$\sigma = \frac{3}{2}\frac{wL}{nbt^2}$

$100 = \frac{3}{2}\frac{2 \times 10^3 * 1}{n * 50 * 10}$

$$100 = \frac{60}{n}$$

$$N = 6$$

$$\delta = \frac{L^2}{4} \frac{\sigma}{Et}$$

$$\delta = \frac{(1000)^2}{4} * \frac{100}{2.1 \times 10^5 \times 10}$$

$$\delta = 11.9 \text{ mm}$$

Helical spring

Generally these springs are helicatin shape. These type of springs are again classified into 2 types

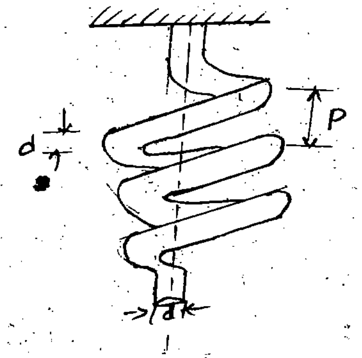
1. closed coiled helical springs.
2. open coiled helical spring.

closed coiled helical spring

The spring in which helix angle is very small or in other words the pitch b/w the adjecent tam is in very small as shown in fig.

As the helix angle is small the bending effect on the spring is ignored and we assume that, the Coils of closed coiled helical springs are to stand purely torsional stresses.

Expression for max. ... the spring



let us consider a closed helical spring as shown in fig,

let, d = Dia of the wire spring

p = pitch of helical spring

n = no. of coils

R = mean radius of the coil.

W = axial load on the spring.

C = Shear modulus

τ = Shear stress induced in the wire

θ = angle of Twist

l = length of wire

w.k.t $T = \frac{\pi}{16} \tau D^3$ — for shaft

but T = Force * distance

Force * distance $= \frac{\pi}{16} * \tau * D^3$

$$W * R = \frac{\pi}{16} * \tau * D^3$$

$$\tau = \frac{16WR}{\pi D^3} = \frac{16WR}{\pi d^3}$$

Expression for deflection in helical spring

workdone by the spring = Avg load * deflection

$$= \frac{W}{2} * \delta$$

strain energy stored = work done.

$$U = \frac{W}{2} * \delta$$

$$\frac{\tau^2}{4C} * V = \frac{W}{2} * \delta$$

$$\left(\frac{16WR}{\pi d^3}\right)^2 * \frac{1}{4C} * \text{Area} * \text{length} = \frac{W}{2} * \delta$$

$\therefore$ length $= 2\pi R$ $\quad$ ($\because$ for n no. of spring $= n\,2\pi R$)

$$\text{Area} = \frac{\pi d^2}{4}$$

$$\frac{16 * 16 W^2 * R^2}{\pi^2 d^6} * \frac{1}{4C} * \frac{\pi d^2}{4} * 2\pi R \cdot n = \frac{W}{2} * \delta$$

$$\delta = \frac{64 W R^3 n}{C d^4}$$

Expression for stiffness of the spring

stiffness is denoted "s".

S = load per unit deflection

$$= \frac{W}{\delta}$$

$$= \frac{W}{\frac{64 W R^3 n}{C d^4}}$$

$$\therefore S = \frac{C d^4}{64 R^3 n}$$

1. A closely coiled helical spring is to carry a load of 500 N, it's mean coil dia is 10 times that of wire dia. Calculate the dia's of these wires, If max shear stress in the material is 80 N/mm². If the stiffness of the spring is 20 N/mm deflection and modulus of rigidity is 8.4 * 10⁴ N/mm². Find the no. of coils in the closely coiled helical spring.

Sol: Given data,

$W = 500$ N

$\tau = 80$ N/mm²

$S = 20$ N/mm

$C = 8.4 * 10^4$ N/mm²

Mean coil is 10 times of wire Dia

Let D = coil dia

d = wire dia.

$D = 10d$

$$\tau = \frac{16 WR}{\pi d^3}$$

$$80 = \frac{16 * 500 * \frac{D}{2}}{\pi d^3}$$

$$80 = \frac{4000 D}{\pi d^3} = \frac{4000 * 10(d)}{\pi (d)^{3\,2}}$$

$$80 = \frac{12732.39}{d^2}$$

$d^2 = 159.15$

$d = 12.62$ mm

$D = 10 * d = 10 * 12.62 = 126.2$ mm

$\delta = \frac{W}{S}$

$= \frac{500}{20} = 25$

$\delta = \frac{64 W R^3 n}{C d^4}$

$25 = \frac{64 * 500 \left(\frac{126.15}{2}\right)^3 n}{8.4 * 10^4 \; (12.62)^4}$

$n =$ 0.